

# Soil improvement with vibrated stone columns — influence of pressure level and relative density on friction angle

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**ABSTRACT:** The application of gravel for improvement of soft soils with pile-like vibrated stone columns has attracted a considerable attention during the last decade. For the design of such a soil improvement, friction angle of the fill material depends markedly on pressure level and relative density. Additionally, size, distribution and shape of grains, together with their mineralogical composition also influence the mechanical behaviour of the soil skeleton in the stone columns. A review of own and published experimental data on gravel and rockfill materials shows that densely prepared specimens yield in most cases very high friction angles lying above  $50^\circ$  at common normal stresses. Thus, conventional design values of  $40^\circ$  are usually too conservative. Furthermore, it is reported on direct and indirect measurements of density inside stone columns installed in situ and on small-scale model tests of stone columns in the laboratory. By exposing vibrated stone columns produced by the Keller method it could be shown that their in situ void ratio corresponds to the maximum density represented by  $e_{min}$  according to DIN 18126.

## 1 INTRODUCTION

The application of gravel as a construction material in geotechnical engineering is manifold. Primarily angular grains of crushed rock are being used as a rockfill in large earth structures like gravity dams or road embankments. During recent years, soil improvement with pile-like vibrated stone columns has also attracted a considerable attention. In the latter case, friction angle of a fill material (rockfill, angular or sub-rounded gravel) plays the crucial role in the calculation of bearing capacity of such stone columns. In case of coarse grained soils, their friction angle depends markedly on pressure level and relative density. Additionally, size, distribution and shape of grains, together with their mineralogical composition also influence the mechanical behaviour.

Experimental investigations in large-scale tests are difficult and rare. Therefore, one often assumes a similar shear strength for gravel like for medium dense sand. A pressure- and density-dependence of friction angle as well as the role of grain properties are usually neglected. This results in conservative values of

the shear strength which contradict the observed real behaviour. In this contribution, a review of published experimental data on gravel and rockfill is supplemented by own test results. Additionally, it is reported on measurements of density inside stone columns in situ and on small-scale model tests of stone columns in the laboratory.

## 2 MECHANICAL BEHAVIOUR OF GRAVEL

*Gravel* usually denotes a coarse grained mineral material which originates from river (alluvial) or slope (debris) deposits, or which is alternatively produced from a crushed rock. This definition of gravel can be also found in Brockhaus or Meyers encyclopedia. The physico-chemical surface and interaction effects (e.g. capillarity) play in case of gravel only a negligible role. The mechanical behaviour is determined solely by geometrical and mineralogical properties of single grains and their distribution in a grain skeleton.

The density limits are substantially influenced by grain size distribution and grain shape (Youd 1973). Both limit void ratios,  $e_{max}$  and  $e_{min}$ , decrease with

increasing non-uniformity coefficient and increasing grain sphericity. The difference between  $e_{max}$  and  $e_{min}$  depends only insignificantly on grain shape (Dickin 1973) and can be considered roughly as  $e_{max} = 1.6e_{min}$  (Miura et al. 1997). An estimation of  $e_{max}$  and  $e_{min}$ , respectively, can be based on grain shape and non-uniformity coefficient using figures published by Youd (Youd 1973).

The granulometric properties of gravel are also important with respect to segregation effects during the construction of stone columns. Well graded gravels and angular grains (crushed rock, rockfill) are more susceptible to segregation than uniformly graded gravels and/or rounded grains (Saucke et al. 1999).

An example of the results from direct shear tests with gravel, which has been used for vibrated stone columns, is shown in Figure 1. The treated gravel from Kerspleben can be characterized as a river gravel 2/32. Rounded up to subrounded grains have a mean grain diameter of  $d_{50} = 17.4$  mm and a non-uniformity coefficient of  $C_u = 2.95$ . The density limits can be described by the limit void ratios  $e_{max} = 0.635$  and  $e_{min} = 0.416$ . Shear tests in the laboratory were performed in a large shear box (manufacturer Wille-Geotechnik) with a specimen area of  $30 \times 30$  cm and a specimen height of approximately 16 cm.

The evaluation of experiments yields a peak friction angle of  $49.3^\circ$  and a residual friction angle of  $45.5^\circ$ . These values correspond well to other results published elsewhere, see (Yasuda et al. 1997) in Table 2. It can be noticed that an increasing normal pressure suppresses dilatancy which is reflected in lower friction angles as well.

Further results from direct shear tests performed at TU Dresden are summarized in Table 1. The evaluation of stress-dependent friction angles from the equation  $\varphi = \arctan(\tau_{max}/\sigma)$  follows from the assumption of a vanishing cohesion. Since the friction angle usually decreases with increasing mean pressure,  $\varphi_{max}$  and  $\varphi_{min}$  values in Table 1 should be related to the pressure limits  $\sigma_{min}$  and  $\sigma_{max}$ , respectively. A conventional interpretation, based on linear regression through three  $(\sigma, \tau)_{max}$  points, is denoted by  $\bar{\varphi}$ .

An overview of friction angles for gravel and rockfill extracted from various laboratory results published in the literature gives Table 2.

The notion  $\sigma$  corresponds either to the normal stress in direct shear tests (DS) oder to the cell pressure in triaxial tests (TX). The tested specimens were prepared at medium to high relative density.

The experimentally determined friction angles reach high values at low mean pressures. Particularly for angular gravel grains (rockfill), these high friction angles rapidly diminish due to grain crushing at increasing pressures. Consequently, limit stress en-

Table 1. Overview of stress-dependent friction angles from direct shear tests with dense gravel for vibrated stone columns.

$(\sigma_{min} = 50 \text{ kN/m}^2, \sigma_{max} = 200 \text{ kN/m}^2)$				
Sample	$\varphi_{max}$ ( $^\circ$ )	$\varphi_{min}$ ( $^\circ$ )	$\bar{\varphi}$ ( $^\circ$ )	Remark
Bahreïn	63.1	53.8	49.3	crushed limestone
Kerspleben	58.8	51.9	49.3	river gravel
mine	57.1	50.9	48.1	subrounded gravel
Lechstaustufe				$C_u = 2.6$
mine	59.2	53.2	49.3	subrounded gravel
Lechstaustufe				$C_u = 2.1$
mine Merchingen	60.4	55.2	53.9	angular grains

Table 2. Various stress-dependent friction angles from published results on shear tests with medium dense to dense gravel.

Gravel	$\varphi_{max}$ ( $^\circ$ )	$\varphi_{min}$ ( $^\circ$ )	Reference Remark
	$\sigma_{min}$ ( $\text{kN/m}^2$ )	$\sigma_{max}$ ( $\text{kN/m}^2$ )	
basalt	60.1	42.2	(Marsal 1967)
	40	1000	TX
river gravel	52.7	42.2	(Marsal 1967)
	40	1000	TX
gneiss	50.8	34.4	(Marsal 1967)
	40	1000	TX, $d_{10} = 6$ mm
dolomite	64.0	43.0	(Raymond et al. 1978)
	15	500	TX, $\gamma = 1.7 \text{ g/cm}^3$
dolomite	54.0	40.0	(Raymond et al. 1978)
	15	500	TX, $\gamma = 1.5 \text{ g/cm}^3$
basalt	64.2	45.6	(Charles and Watts 1980)
	27	695	TX
sandstone	60.1	37.4	(Charles and Watts 1980)
	27	695	TX
various	52.2	45.6	(Hettler 1987)
	30	170	TX, mean values
gabro	53-61	45-51	(Vaughan 1994)
	100	500	RS
gabro	44-49	41-44	(Vaughan 1994)
	100	500	RS, weathered
river gravel	49.6	44.8	(Yasuda et al. 1997)
	50	290	TX, $d_{max} = 63.5$ mm
river gravel	51.3	46.9	(Yasuda et al. 1997)
	100	590	TX, $d_{max} = 174$ mm
various	42-55	36-47	(Leps 1970)
			(Indraratna et al. 1993)
basalt	50	700	TX
	71.8	45.6	(Indraratna et al. 1998)
basalt	8	240	TX, $d_{50} = 30$ mm
	70.0	51.1	(Indraratna et al. 1998)
basalt	8	120	TX, $d_{50} = 39$ mm

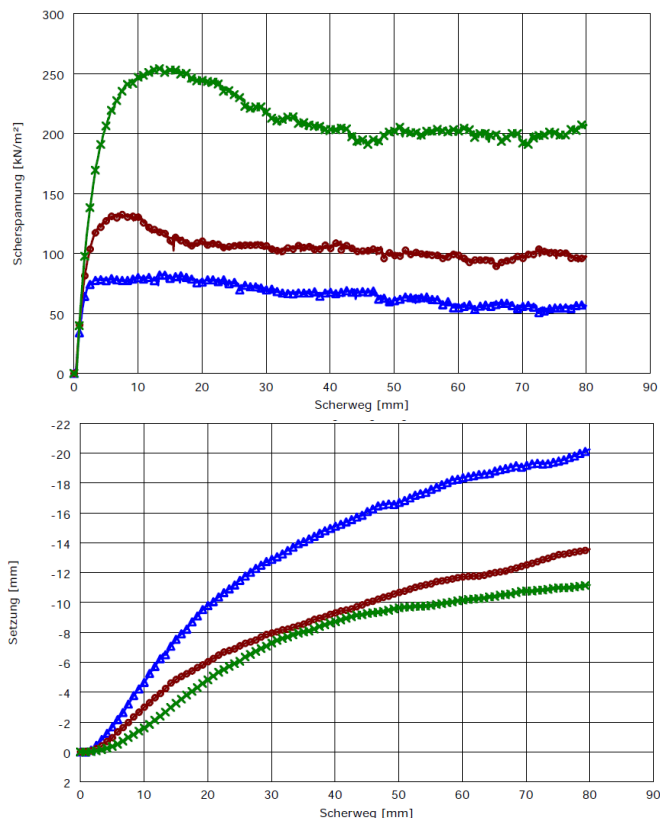


Figure 1. Results of direct shear tests with gravel (normal stresses: 50, 100 und 200 kN/m<sup>2</sup>). Shear stress (top) and vertical displacement (bottom) vs shear displacement.

velopes of Mohr stress circles are strongly curved. It has been recommended (e.g. (Barton and Kjaernsli 1981)) to consider friction angle proportional to  $1/\log \sigma$  to get a crude estimation of the pressure impact. Generally, according to Table 2, the values of friction angle do not drop below 40° for  $\sigma < 500$  kN/m<sup>2</sup>. The only exceptions are gneiss and sandstone rockfills, which are rarely used for construction purposes.

An uniformly graded gravel is more pressure sensitive than a well graded one (Indraratna et al. 1998), i.e. grain crushing accompanied by a decrease of friction angle is more pronounced in the first case. A similar role plays grain size: larger grains are crushed easier than the small ones, therefore the pressure sensitivity increases with a growing grain size (Marsal 1967; Marachi et al. 1972; Indraratna et al. 1998). Nevertheless, in case of river gravel, increasing shear strength with increasing grain size has been also reported (Varadarajan et al. 2006).

The impact of density on the shear strength of gravel has been less investigated yet. Obviously, friction angle decreases with a decreasing density as it has been extensively described for sands and other granular materials, see e.g. (Kolymbas and Wu 1990). Some typical values can be obtained from results on crushed limestone published by Raymond and Davies (1978). Whereas one gets a friction angle of 47° at  $\sigma=100$  kN/m<sup>2</sup> for a density of  $\gamma=1.7$  g/cm<sup>3</sup>, it drops

to  $\varphi=45^\circ$  for  $\gamma=1.6$  g/cm<sup>3</sup>, to  $\varphi=43^\circ$  for  $\gamma=1.5$  g/cm<sup>3</sup> and to  $\varphi=40^\circ$  for  $\gamma=1.4$  g/cm<sup>3</sup>. Further data on density dependence presented e.g. Moroto and Ishii (1990) or Mogami and Yoshikoshi (1968), who used the relationship  $(1 + e) \sin \varphi \approx \text{const.}$  for the evaluation of the test results. One can consider the angle of repose as a lower boundary which corresponds to the pressure-independent critical (residual) friction angle  $\varphi_r$ .

The influence of relative density and mean pressure on the mechanical behaviour of gravel can be well described by hypoplastic constitutive models (Gudehus 1996; von Wolffersdorff 1996). It has been demonstrated that the hypoplastic material parameters can be estimated directly from the granulometric properties and/or more precisely determined with help of basic standard tests in the laboratory (Herle and Gudehus 1999). The calibration of the hypoplastic parameters works also for gravel as it was shown for a crushed limestone (Herle 2000).

### 3 MEASUREMENT OF IN SITU DENSITY

The in situ density of vibrated stone columns is very uncertain. The diameter of columns is known only very roughly which makes the density calculation from the filled gravel mass unreliable. Indirect sounding (dynamic or static penetration test) inside the columns is also hardly successful because of unavailable calibration data for the same material with a known density distribution at particular pressure. Last but not least, the relationships from DIN 4094-3 are not useful for gravel installed by a depth vibrator and they do not properly take into account the influence of mean stress and of many granulometric parameters.

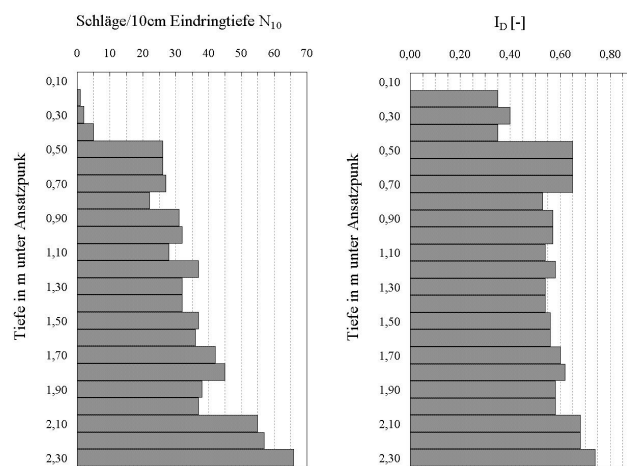


Figure 2. Light dynamic penetration test in a stone column and an interpretation of the results as relative density index  $I_D$  according to DIN 4094-3.

Figure 2 shows the evaluation of a light dynamic penetration test in an upper part of a stone column in Kerspleben close to Erfurt, Germany. The surrounding subsoil has corresponded to clayey silt. The interpretation of the penetration results according to DIN

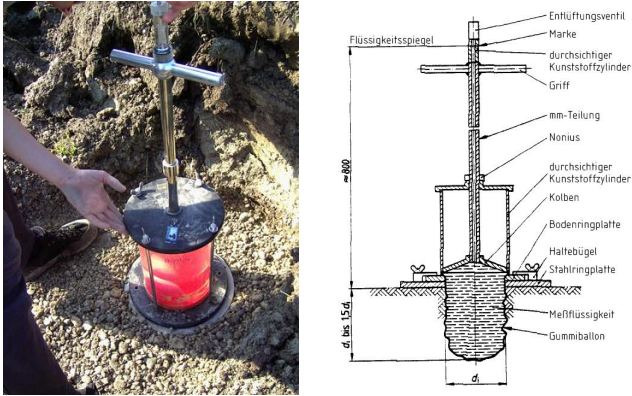


Figure 3. Direct measurement of in situ density with a balloon method according to DIN 18125-2.

4094-3 suggests a medium relative density. Therefore, a direct measurement of density in stone columns has been performed for a check (Hentschel 2005). Upper parts of several columns have been stepwise dug away and their in situ density has been determined with a balloon method according to DIN 18125-2, see Figure 3.

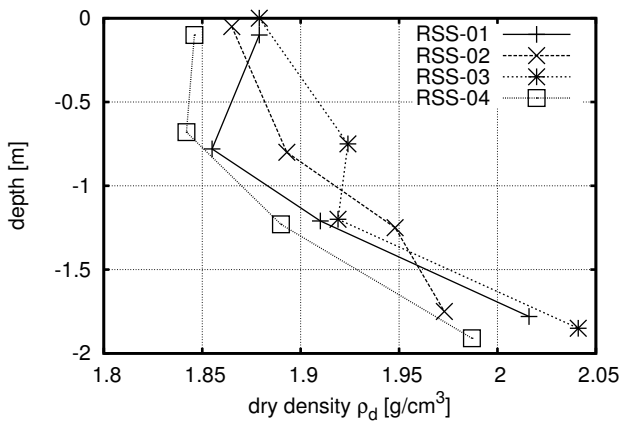


Figure 4. Measured increase of gravel dry density  $\rho_d$  with increasing depth of stone columns.

The measured field densities are summarized in Figure 4. They correspond to void ratios in a range between 0.27 and 0.39. Comparing them with a value of  $e_{min}=0.416$  (see the last section) one can conclude that relative density inside the stone columns is higher than one! This disproves the conventional interpretation according to DIN 4094-3 and points out to an inadmissible application of correlations for indirect field testing.

The measured density inside the stone columns increases significantly with depth, see Figure 4. It has been difficult to explain such a marked densification by a depth vibrator for the used gravel. Therefore, the direct measurement of density for each sample has been supplemented by the determination of grain size distribution and density limits in the laboratory. The evaluation of the grain size distributions has revealed an increase of nonuniformity coefficient with depth,

see Figure 5 (cf. with  $C_u=2.95$  of the gravel in original state). At the same time, the change of nonuniformity is related to a decrease of mean grain size  $d_{50}$ . This means that during densification via depth vibrator a distinct grain crushing takes place, being accompanied by grain segregation (falling through of small grains).

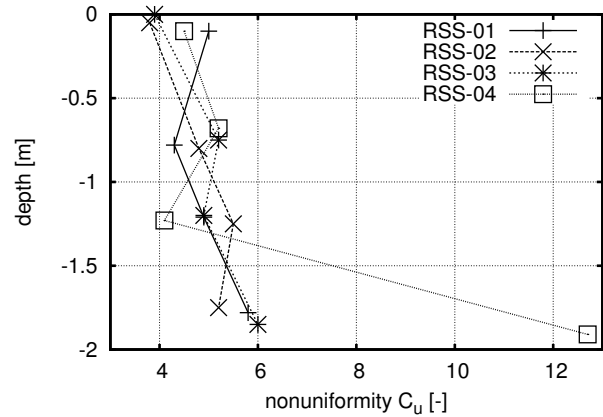


Figure 5. Increase of nonuniformity coefficient  $C_u$  with depth.

It has been noticed in the preceding section that a change of nonuniformity coefficient has a decisive influence on the density limits (Youd 1973).  $e_{min}$  decreases for growing nonuniformity, which has been also observed in our laboratory experiments. Figure 6 depicts a trend of the measured void ratios for increasing depth. It can be clearly seen that the measured void ratios inside the stone column, even at the vicinity of ground surface, correspond to the  $e_{min}$  values from the laboratory, i.e. they confirm the maximum density of gravel in the field.

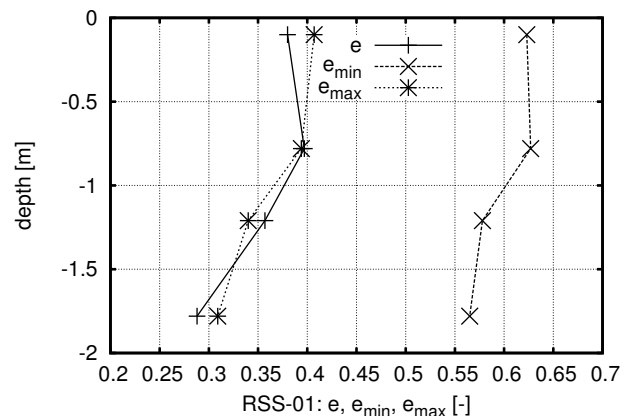


Figure 6. Decrease of limit void ratios and in situ void ratios with increasing depth.

The in situ void ratios measured inside the stone columns are plotted within the diagrams by Youd (Youd 1973) in Figure 7. They are located along the curve for nonuniformity-dependent minimum void

ratio and suggest the grain shape between subangular and subrounded. This is plausible if considering that many grains of gravel has been crushed during the densification by a depth vibrator.

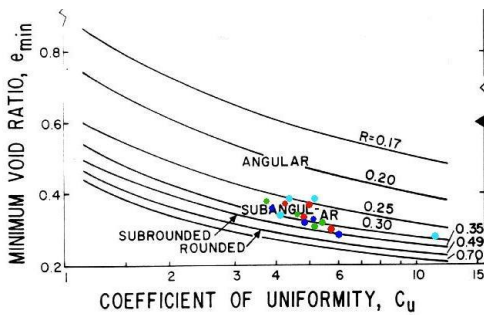


Figure 7. Comparison of in situ void ratios (points) with  $e_{min}$  according to Youd (Youd 1973).

#### 4 MODEL TESTS

In order to answer the question about the influence of relative density on bearing capacity of vibrated stone columns, a series of small scale 1-g model tests has been performed in the laboratory (Hentschel 2005). The stone columns have been represented by a medium quartz sand which has been installed with help of a plastic mould and compacted by tamping using a falling weight. The surrounding soil was modelled by a mixture of sand grains and plastic grains, so-called Soiltron, (Laudahn 2004), which suppressed sand dilatancy due to low pressures at small overburden. A deformation measurement inside the column using the Particle Image Velocimetry (PIV) has been enabled by modelling only of one half of the column.

The vibrated stone columns have been loaded up to the bearing capacity failure. The load has been transferred through a rigid foundation at the ground surface. A typical experimental setup and displacement measurements with PIV are shown in Figure 8 (Herle et al. 2006).

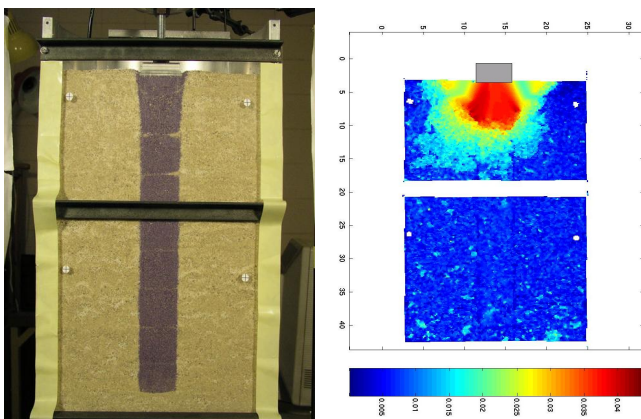


Figure 8. Model tests for loading a single stone column and measurement of displacements with Particle Image Velocimetry.

An example of experimental results is given in Fig-

ure 9. The column with dense sand yields almost a double bearing capacity than a column with loose sand. Moreover, the settlements mobilized at peak are much smaller in the first case. This demonstrates – at least qualitatively – the importance of a maximum densification for the construction of vibrated stone columns.

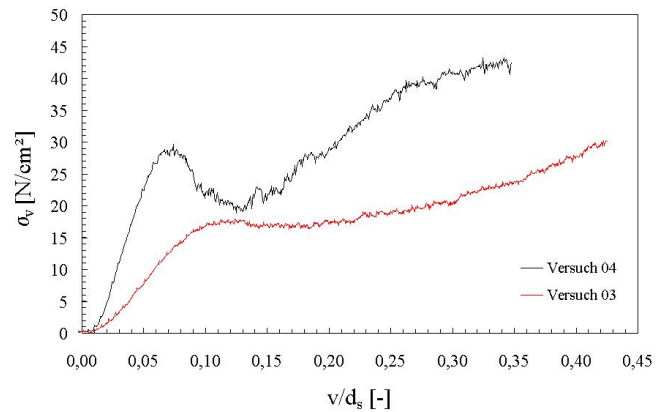


Figure 9. Load-displacement curves from the model tests of a loose (Versuch 03) and a dense (Versuch 04) sand column.

#### 5 CONCLUSIONS

The mechanical behaviour of gravel and rockfill depends in an outstanding manner on mean pressure and relative density. At the same time, granulometric factors like nonuniformity, grain size and grain shape play an important role as well. Large scale shear box tests with densely prepared specimens yield in most cases very high friction angles which lie above  $50^\circ$  at low normal stresses. Thus, conventional design values of  $40^\circ$  are usually too conservative.

By exposing vibrated stone columns produced by the Keller method it could be shown that their in situ void ratio corresponds to the maximum density represented by  $e_{min}$  according to DIN 18126. In course of densification, grain crushing and segregation take place which results in decrease of the limit void ratios. A conventional interpretation of dynamic penetration test according to DIN 4094-3 is not suitable for the determination of density inside vibrated stone columns, if a calibration (e.g. with balloon method) is not available.

#### ACKNOWLEDGEMENT

The field tests have been carried out by M. Hentschel with help of S. Gesellmann. The model tests were performed by M. Hentschel at the Department of Engineering Geology at Charles University Prague. The support of Dr. Jan Boháč is gratefully acknowledged.

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